Name ____ AP Calculus

Summer Reading

There will be a test on this reading material the first week of class!

Over the past few years I've found that a lot of the problems that people have in calculus stem more from problems with algebra and trig than the actual calculus concepts. Here's a review of topics in which students historically have been weak. Make sure you know these ideas both forwards and backwards (that is – I give you either side the equation, and you can give me the other side). If after reading these examples and doing your own independent study (using old notes, the internet, etc) you are still confused about something, **feel free to email me** at svogel@ktufsd.org.

Funky exponents

Negative exponents

Negative exponents basically produce *reciprocals*. Take the reciprocal of the term being "exponented," and then use the positive version of the exponent.

Examples:

1. $2x^{-3} = \dfrac{2}{x^3}$ — note that the power is only on the *x*, not the 2

2. $(2x)^{-3} = \dfrac{1}{(2x)^3}$ — this time the entire 2*x* is being raised to the power

3. $\dfrac{4}{x^{-5}} = 4x^5$ — negative exponents in the denominator would bring the variable *up*

Fraction exponents

Fraction exponents produce *roots*. The denominator of the fraction is the type of root involved. The numerator of the fraction is the power on the whole term.

Examples:

1. $x^{1/2} = \sqrt[2]{x}$ — (not that you must write the little 2 in the root)

2. $x^{1/3} = \sqrt[3]{x}$ — (okay – this time you need to write the 3 ☺)

3. $x^{3/5} = \sqrt[5]{x^3}$ *or* $\left(\sqrt[5]{x}\right)^3$ — either version is correct

Negative fraction exponents –

Just put the two previous ideas together!

Example: $2x^{-1/4} = \frac{2}{\sqrt[4]{x}}$

Zeros in the numerator and/or denominator of fractions

- A fraction with zero *only in the numerator* is equal to **zero**.

  Examples:

  1. $\frac{0}{5} = 0$
  2. $\frac{x-5}{x+6} = 0$ when x = 5, since that will cause a zero in the numerator.

- A fraction with zero *only in the denominator* is **undefined** (its value approaches $\pm\infty$)

  Examples:

  1. $\frac{5}{0} = \mathit{undefined}$
  2. $\frac{3}{x^2-4}$ is undefined for x = ±2

- A fraction with zero in the numerator *and* denominator is **indeterminate** (which means we probably have more work to do)

  Example: $\frac{x+3}{x^2-9}$ is undefined for x = 3, and indeterminate for x = -3

Natural logs – logarithms with base *e*

$\ln(ab) = \ln(a) + \ln(b)$

$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

$\ln a^b = b\ln(a)$

$\ln e = 1$

$\ln 1 = 0$

Solving inequalities

Usually the easiest way to solve an inequality is to "pretend it is equal." Find the places where the *equality* is true, and then test nearby *x*-values to see where the *inequality* is true.

Example:	$x^2 - 4 > 0$	
	$x^2 - 4 = 0$	**pretend** that we really have an equal sign
	$(x+2)(x-2) = 0$	**solve**, in this case by factoring
	$x = \pm 2$	so we need to check around 2 and -2

$x^2 - 4$	21 true		-4 false		21 true
x	**-5**	-2	**0**	2	**5**

test **nearby numbers** by plugging them into the original inequality

so the final answer is ***x* < -2 or *x* > 2**

Piecewise-defined functions

Piecewise functions are defined by multiple expressions, each one applying to a different set of *x*-values.

Example: $f(x) = \begin{cases} x^2 - 2, & x < 1 \\ 3x - 1, & x \geq 1 \end{cases}$

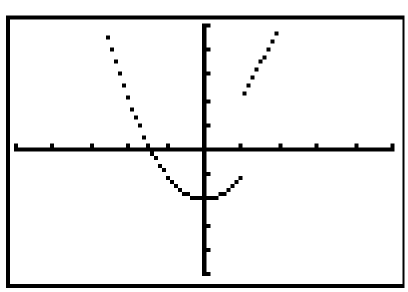
This function is saying "when x is less than one, the function acts like $f(x) = x^2 - 2$; when x is greater than or equal to 1, the function acts like $f(x) = 3x - 1$."

We can type these into the calculator like this:

$Y_1 = (x^2 - 2)(x < 1) + (3x - 1)(x \geq 1)$ [the inequality signs are in ]

The graph would look like this →

(note that I'm using dotted mode; if I used connected mode we may not see the jump discontinuity at $x = 1$, it depends on which calculator you have)



Trig

Things to remember:

SOH-CAH-TOA $\sin x = \dfrac{1}{\csc x}$ $\cos x = \dfrac{1}{\sec x}$ $\tan x = \dfrac{1}{\cot x}$

$\sin^2 x + \cos^2 x = 1$ $\sin(2x) = 2\sin x \cos x$

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan x$	0	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef'd

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	1	0	-1	0
$\cos x$	1	0	-1	0	1
$\tan x$	0	undef'd	0	undef'd	0

Note that all the angles are in radian measure. We will always use radian measure in this class.

Simplifying radicals, fractions, exponents, etc

Radicals and powers do not "distribute" over addition or subtraction.
They DO "distribute" with division and multiplication

Examples:

1. $\sqrt{4-x^2}$ is NOT equal to $2 - x$, however $\sqrt{4x^2} = 2x$
2. $(y + 3)^2$ is NOT equal to $y^2 + 9$, however $(3y)^2 = 3^2y^2 = 9y^2$
3. $\dfrac{2a+b}{b}$ is NOT equal to $2a$ (or $2a + 1$ either!), however $\dfrac{2ab}{b} = 2a$
4. $\text{Anything}^0 = 1$ (except 0^0 – that's another indeterminate form)

Also – don't forget your order of operations!

Example: Simplify $3(2x-5)^2$

Correct:

$$3(2x-5)^2 = 3(4x^2-20x+25)$$
$$= 12x^2 - 60x + 75$$

Incorrect:

$$3(2x-5)^2 = (6x-75)^2$$
$$= 36x^2 - 900x + 5625$$

Functions

$f(x) = x^2 + 5x$ is a function named *f*, with input *x*; it's NOT "*f* times *x*"

Given that function, we could say:

$f(t) = (t)^2 + 5(t)$ note that we simply replaced the *x* in the original with *t*.

$f(calculus) = (calculus)^2 + 5(calculus)$ same idea

$f(x+h) = (x+h)^2 + 5(x+h)$ replace the original *x* with *x* + *h*.

If function notation throws you for a loop, perhaps you should rewrite the function with blank parenthesis in place of the *x*. Then whatever the problem has inside the parenthesis after *f*, put it in all the parenthesis.

Example: to do *f* (*abc)*, start with:

$$f(\quad) = (\quad)^2 + 5(\quad)$$

then…

$$f(abc) = (abc)^2 + 5(abc)$$

Also, remember that *f(x)* is the same as *y*, so if someone tells you that $f(2) = 5$, then you know that the point (2, 5) is part of the graph.

Even & Odd

Even functions are symmetric in the *y*-axis, odd functions have rotational symmetry in the origin.